A Generic Noise Model for InSAR Time Series Based on Stepwise Error Propagation

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Fringe 2021 Recommendations

Communicating results and their uncertainty
the key recommendations in this regard are:

1. Standardization of output products
2. Error-bars and easy-to-understand statistics should accompany results and derived products for more rigorous analysis.

Importance of Noise Model for InSAR

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   How precise are the InSAR products: deformation rate, timeseries, etc ? How to estimate correct error-bars?

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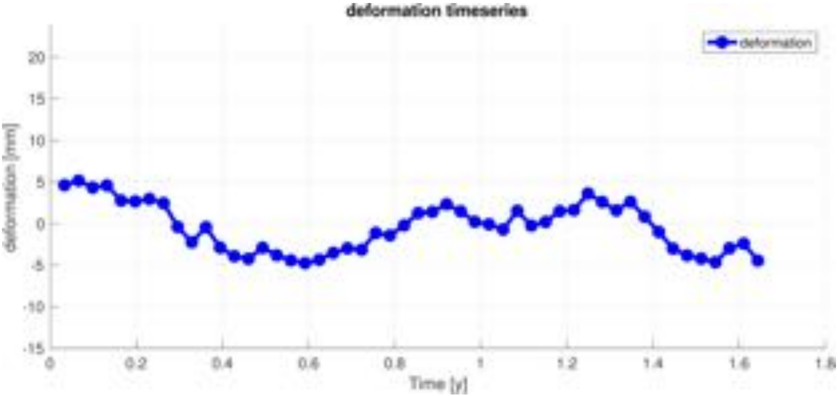
   Exploitation of the inverse of the covariance matrix as a weight matrix provides estimators with desired optimality aspects: minimum variance, maximum likelihood.
3. **Proper Noise Filtering**
   How to optimally separate signals (deformation) from noise (atmosphere, scattering, etc.)
4. **Proper interpretation**
   To decide whether the InSAR-based deformation estimates are significant or not? (Difficult in case of correlated noise)

Importance of Noise Models for InSAR

Interpretation: Deformation or Noise?
Example 1:



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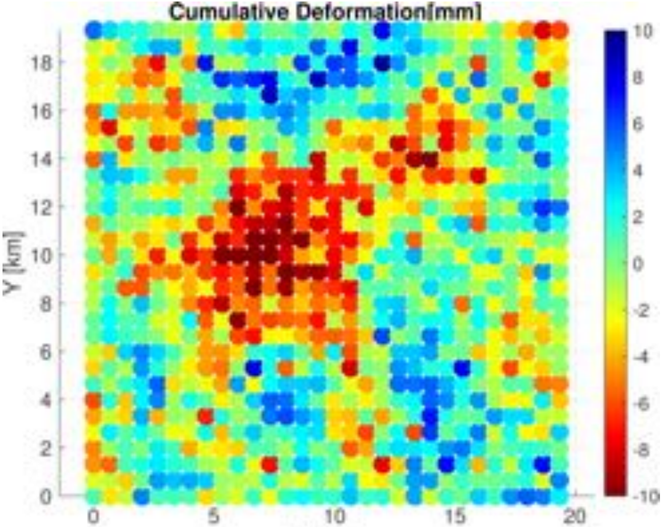
Interpretation: Deformation or Noise?

Example 2:



Importance of Noise Models for InSAR

Two Aspects of Quality Description

1. **Precision**
   What is the dispersion around the mean value? How wide shoule be the error-bars?
2. **Accuracy**
   How close are the measurments to their true value? Effect of Unwarpping errors!

The main goal of this study is to describe the **precision** of InSAR timeseries in a generic from

For Accuracy and quality of Unwarpping, see our poster No.83

InSAR Generic Noise Model: What do we already know?

A good body of knowledge of InSAR noise components is already exist

1. **Coherence_affected Noise**
   Goodman, 1976; Madsen, 1986; Bamler and Hartl, 1998, Hanssen 2001, and many other studies
2. **Atmospheric Noise**
   Hanssen 2001, Liu, 2012, and many other studies
3. **Coherence_affected and atmosphere in timeseries**
   Monti-Guarnieri and Tebaldini, (2007)
   Hybrid Cramer-Rao bound for InSAR timeseries
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But, are these models representative for final InSAR timeseries/products delivered by different algorithms/softwares?

InSAR Generic Noise Model: What is the problem?

Processing-induced noise?

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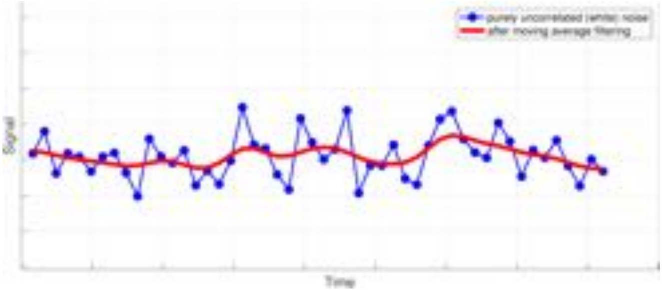

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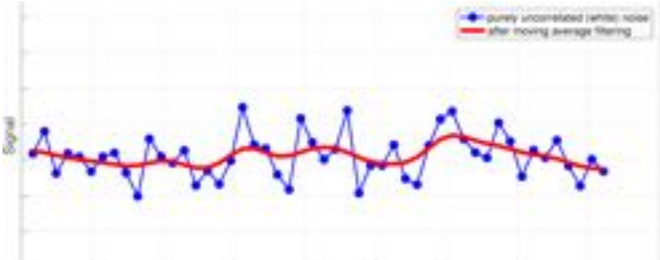

Currently available noise models are not representative of true noise structure in final InSAR timeseries

InSAR Generic Noise Model: Influencing Factors

Noise structure of final InSAR timeseries depends on :

1. Scattering characterstics of the targets (via coherence matrix or amplitude stability)
2. Atmospheric delay heterogeneity (via atmopheric noise models)
3. Data characteristics (e.g., multilooking factor, number of acquisitions, revisit time)
4. Type of deformation mechanism and its spatio-temporal behavior
5. Temporal/Spatial kernels used for noise filtering
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We need a generic noise model to account for all these factors

InSAR Generic Noise Model: Uncertainty Propagation

- **Uncertainty Propagation**
  To derive or assume a noise structure for raw InSAR timeseries followed by an error propagation through all the processing steps.

Challenges of this approach

- Complexity of the processing steps
- Incomparable strategies/algorithms among InSAR methodologies,
- Large volume of spatio-temporal InSAR observations

InSAR Generic Noise Model: Objectives

- To develop a methodology to propagate all the noise sources
- To design an efficient error propagation scheme through all the processing steps
- The main focus on: Noise covariance matrix (not only variances)
- Large volume of data: to derive an **analytical closed-form expression** to reconstruct the variances and covariances for any given spatio-temporal deformation estimate.

InSAR Generic Noise Model: Steps

Steps:

1. Exploiting the existing body of knowledge about InSAR noise sources to propagate errors through the interferogram generation steps (i.e., from statistics of SAR data to statistics of interferometric phase time series)
2. Further error propagation in the fltering step.

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For the latter, we formulate the filtering step in a mathematical framework based on the prediction theory (Least-squares Collocation or Wiener Filtering). This mathematical formulation is:

- **Generic:** It can cover different existing methodologies/algorithms
- **Flexible:** It can digest deterministic/stochastic assumptions about spatiotemporal behavior of different signal/noise components
- **Simple:** It allows the application of the linear error propagation

InSAR Generic Noise Model: Constructing the Mathematical Model

Generic form of the prediction model:

$$y = \boxed{\underbrace{Ax}_{\text{functional model}}} + \boxed{\underbrace{s}_{\text{Unmodeled Deformation}}} + \boxed{\underbrace{a}_{\text{Atmosphere}}} + \boxed{\underbrace{n}_{\text{Coherence\_based noise}}}$$

- y : timeseries of a PS/DS or timeseries of set of PS/DS or set of phase values over a single interferogram
- Ax : any spatial or temporal trend in the data (e.g. linear model, planar spatial trend, etc.)
- s : deformation not captured by Ax (described stochastically)
- a : atmospheric signal, mainly turbulence (described stochastically)
- n : scattering, thermal, coregistration/respampling noise components (described stochastically)

$$\boxed{Q_y = Q_s + Q_a + Q_n}$$

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$$z = \boxed{\underbrace{A_z x}_{\text{Functional model}}} + \boxed{\underbrace{s_z}_{\text{Unmodeled Deformation}}}$$

- z : final filtered timeseries of a PS/DS or a set of PS/DS
- Ax : any spatial or temporal deformation trend in z
- s : deformation not captured by $A_z x$

InSAR Generic Noise Model: Constructing the Mathematical Model

$$y = Ax + s + a + n \qquad z = A_z x + s_z$$

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Generic solution for z

$$\text{step 1}: \hat{x} = \underbrace{(A^T W A)^T A^T W}_{R} y \quad \text{(Least Squares Estimate of x)}$$

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$$\text{step } 2: \; \hat{z} = A_z \hat{x} + K(y - A\hat{x}) \quad \text{(Prediction of z)}$$

$$\hat{z} = A_z R y + K(y - ARy) = \boxed{\underbrace{(A_z R + K - AR)}_{F}} \, y$$

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Having F and Q_y then we can obtain the covaraince matrix of $\hat{z}$:

$$\boxed{Q_{\hat{z}} = F Q_y F^T}$$

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How to get Q_y ?

InSAR Generic Noise Model: Constructing Qy

Steps:

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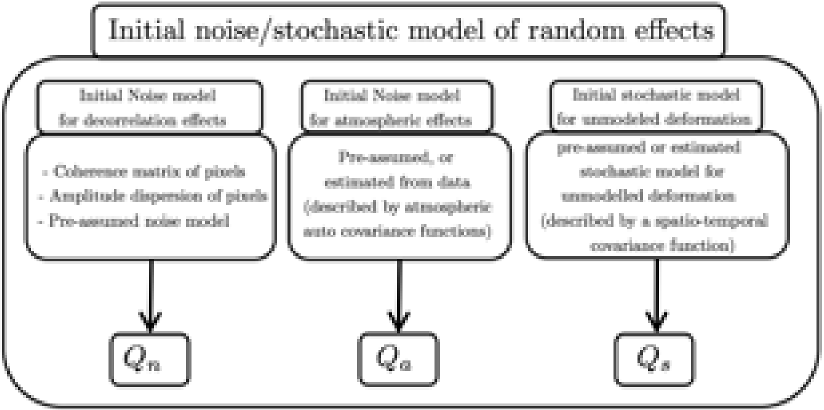

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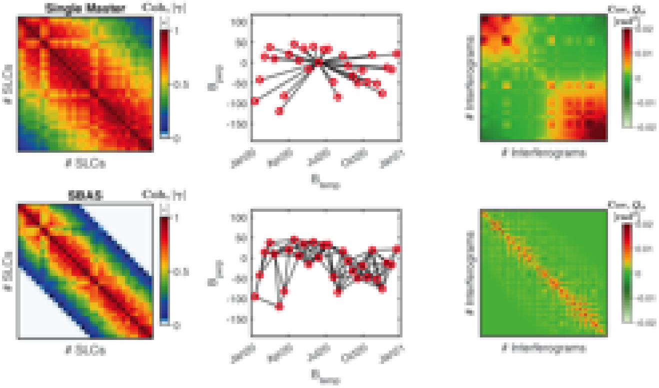

Samie-Esfahnay and Hanssen, 2017, On the evaluation of second order phase statistics in SAR interferogram stacks

InSAR Generic Noise Model: Challenges

Challenges of the full error propagation: $Q_{\hat{z}} = FQ_yF^T$

1. There are some filtering steps applied in the space domain and some in the time domain
2. The error propagation should capable to join the spatial and the temporal operations

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Solution:

1. Applying a step-wise error propagation, first in time then in space
2. Merging the temporal and spatial models together by a simple transformation between the time domain and the space domain

InSAR Generic Noise Model: Time+Space Filtering

Steps:

1. Apply a high-pass filter in the time domain on timeseries of set of initial points (we call it set N) to separate deformation from atmosphere/noise
2. Apply a low-pass filter on the set N in each interferogram to estimate the spatially low frequency noise (mainly atmospheric noise)
3. Interpolate the estimated noise on other set of points (we call it set P) to get the correction
4. Apply the correction on the phases of set P

InSAR Generic Noise Model: Time Domain Propagation

Mathematical model for pixel i :

$$\left[y_i\right]_{N\times 1} = A_i\, x_i + s_i + \underbrace{a_i + n_i}_{e_i} \qquad \begin{cases} y_i & N\times 1 \text{ phase vector of pixel } i \\ A_i x_i & \text{Temporal trend in timeseries of pixel } i \\ s_i & N\times 1 \text{ phase vector of unmodeled defo} \\ e_i & N\times 1 \text{ phase vector of noise components} \end{cases}$$

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Solution for e_i (high-pass filter) :

$$\begin{cases} \hat{x}_i = (A_i^T W_i A_i)^{-1} A_i^T W_i y_i \\ \hat{e}_i = K_i (y_i - A_i \hat{x}_i) = \\ \qquad K_i \underbrace{(I_N - A_i A_i^T W_i A_i)^{-1} A_i^T W_i)}_{R_i} y_i \end{cases}$$

$$\begin{cases} W_i & N\times N \text{weight matrix} i \\ K_i & N\times N \text{ high-pass kernel} \end{cases}$$

$$\boxed{\hat{e}_i = K_i R_i\; y_i}$$

InSAR Generic Noise Model: Time Domain Propagation

Mathematical model for pixel i :

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Mathematical model for ...
set of M pixels in an initial Network (denotes by N) :

$$\underbrace{\begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \vdots \\ \hat{e}_M \end{bmatrix}}_{\hat{e}_\mathrm{N}} = \underbrace{\begin{bmatrix} K_1 & & & \\ & K_2 & & \\ & & \ddots & \\ & & & K_M \end{bmatrix}}_{K} \underbrace{\begin{bmatrix} R_1 & & & \\ & R_2 & & \\ & & \ddots & \\ & & & R_M \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix}}_{y_\mathrm{N}}$$

$$\hat{e}_\mathrm{N} = KR \; y_\mathrm{N}$$

InSAR Generic Noise Model: Space Domain Propagation

Mathematical model for the network points in kth interferogram :

$$\underbrace{\begin{bmatrix} \hat{e}_1^k \\ \hat{e}_2^k \\ \vdots \\ \hat{e}_M^k \end{bmatrix}}_{\hat{e}_N^k} = B_N^k z^k + a^k + n^k \qquad \begin{cases} \hat{e}_N^k & \text{vector of estimated } e \text{components} \\ B_N^k & \text{Spatial trend in } k\text{th ifg} \\ a^k & M \times 1 vectorof \text{ atmospheric noise } k\text{th ifg} \\ n^k & M \times 1 \text{ coh\_based noise } k\text{th ifg} \end{cases}$$

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Solution for a_k for set of p pixels in P in kth interferogram (low pass filter):

$$\begin{cases} \hat{z}^k = ({B_N^k}^T W^k B_N^k)^{-1} {B_N^k}^T W^k \hat{e}_N^k \\ \hat{a}_P^k = B_P^k \hat{z}^k + L^k(\hat{e}_N^k - B_N^k \hat{z}^k) = H^k \hat{e}_N^k \end{cases} \qquad \begin{cases} W^k & M \times M \text{ weight matrix} i \\ L^k & p \times M \text{ high-pass kernel} \end{cases}$$

$$\boxed{\hat{a}_P^k = H^k \hat{e}_N^k}$$

InSAR Generic Noise Model: Space-Time Connection

$$\underbrace{\hat{e}_{\mathrm{N}} = KR\ y_{\mathrm{N}}}_{\text{Time Domain Propagation}} \qquad \underbrace{\hat{a}_{\mathrm{P}}^{k} = H^{k}\hat{e}_{\mathrm{N}}^{k}}_{\text{Space Domain Propagation}}$$

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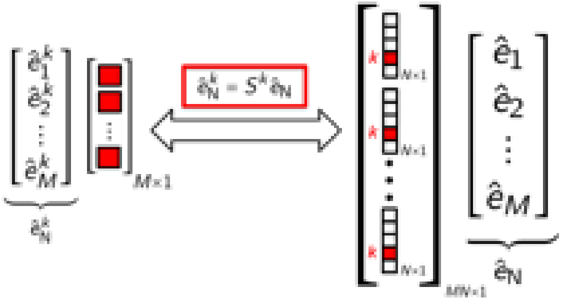
$$\underbrace{\hat{e}_N = KR\, y_N}_{\text{Time Domain Propagation}} \qquad \underbrace{\hat{a}_P^k = H^k \hat{e}_N^k}_{\text{Space Domain Propagation}}$$

To merge these two equations, we should write $\hat{e}_N^k$ as a function of $\hat{e}_N$



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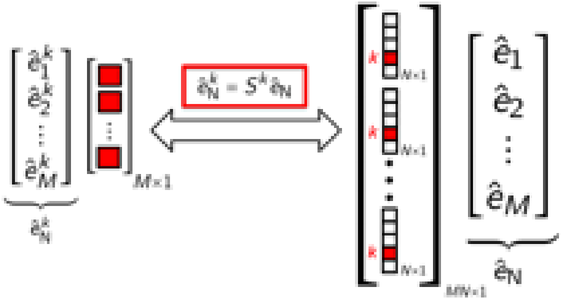

Merging all equations above, we get:

$$\boxed{\hat{a}_P^k = H^k S^k KR\ y_N}$$

InSAR Generic Noise Model: Final filtered phases

$$\hat{a}_{\mathrm{P}}^{k} = \underbrace{H^k S^k K R}_{F^k}\ y_{\mathrm{N}}$$

Above equation is the correction that shoud be applied on phase values of set P in the *kth* interferogram.
So the filtered phases of points in P in the kth and fth interferograms are:

$$\begin{matrix} \hat{y}_{\mathrm{P}^k} = y_{\mathrm{P}}^{k} - F^k y_{\mathrm{N}} \\ \hat{y}_{\mathrm{P}^f} = y_{\mathrm{P}}^{f} - F^f y_{\mathrm{N}} \end{matrix} \Longrightarrow \begin{bmatrix} \hat{y}_{\mathrm{P}^k} \\ \hat{y}_{\mathrm{P}^f} \end{bmatrix} = \begin{bmatrix} I_p & 0 & F^k \\ 0 & I_f & F^f \end{bmatrix} \begin{bmatrix} y_{\mathrm{P}^k} \\ y_{\mathrm{P}^f} \\ y_{\mathrm{N}} \end{bmatrix}$$

The final filtered phases were written
as a linear function of original timeseries

InSAR Generic Noise Model: precision of filtered phases

Precision of filtered phases is described by the dispersion of the remaining noise sources in the filtered data:

$$\begin{array}{l}\epsilon_{\mathrm{P}}^{k} = e_{\mathrm{P}}^{k} - F^{k} y_{\mathrm{N}} \\ \epsilon_{\mathrm{P}}^{f} = e_{\mathrm{P}}^{f} - F^{f} y_{\mathrm{N}}\end{array} \Longrightarrow \begin{bmatrix} \epsilon_{\mathrm{P}^{k}} \\ \epsilon_{\mathrm{P}^{f}} \end{bmatrix} = \underbrace{\begin{bmatrix} I & 0 & F^{k} \\ 0 & I & F^{f} \end{bmatrix}}_{F} \begin{bmatrix} e_{\mathrm{P}}^{k} \\ e_{\mathrm{P}}^{f} \\ y_{\mathrm{N}} \end{bmatrix}$$

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Applying linear (co)variance propagation law:

$$\begin{bmatrix} Q_{\epsilon_{\mathrm{P}}^{k}} & Q_{\epsilon_{\mathrm{P}}^{k}\epsilon_{\mathrm{P}}^{f}} \\ Q_{\epsilon_{\mathrm{P}}^{f}\epsilon_{\mathrm{P}}^{k}} & Q_{\epsilon_{\mathrm{P}}^{f}} \end{bmatrix} = F \begin{bmatrix} Q_{e_{\mathrm{P}}^{k}} & Q_{e_{\mathrm{P}}^{k} e_{\mathrm{P}}^{f}} & Q_{e_{\mathrm{P}}^{k} y_{\mathrm{N}}} \\ Q_{e_{\mathrm{P}}^{f} e_{\mathrm{P}}^{k}} & Q_{e_{\mathrm{P}}^{f}} & Q_{e_{\mathrm{P}}^{f} y_{\mathrm{N}}} \\ Q_{e_{\mathrm{N}} y_{\mathrm{P}}^{k}} & Q_{e_{\mathrm{N}} y_{\mathrm{P}}^{f}} & Q_{y_{\mathrm{N}}} \end{bmatrix} F^{T}$$

InSAR Generic Noise Model: precision of filtered phases

(Co)variance matrix of phase values of points P in kth ifg:

$$Q_{\epsilon_{\mathrm{P}}^k} = Q_{e_{\mathrm{P}}^k} - F^k Q_{y_{\mathrm{N}} e_{\mathrm{P}}^k} - Q_{e_{\mathrm{P}}^k y_{\mathrm{N}}} {F^k}^T + F^k Q_{y_{\mathrm{N}}} {F^k}^T$$

Cross (co)variance between kth and fth ifg:

$$Q_{\epsilon_{\mathrm{P}}^k \epsilon_{\mathrm{P}}^f} = Q_{e_{\mathrm{P}}^k e_{\mathrm{P}}^f} - F^k Q_{y_{\mathrm{N}} e_{\mathrm{P}}^f} - Q_{e_{\mathrm{P}}^k y_{\mathrm{N}}} {F^f}^T + F^k Q_{y_{\mathrm{N}}} {F^f}^T$$

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1. With the first equation we can compute the variance of any target in any interferogram
2. With the second equation, we can compute the covariance between any two phase values in the time/space domains

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$$Q_{\epsilon_{\mathrm{P}}^k} = Q_{e_{\mathrm{P}}^k} - F^k Q_{y_{\mathrm{N}} e_{\mathrm{P}}^k} - Q_{e_{\mathrm{P}}^k y_{\mathrm{N}}} {F^k}^T + F^k Q_{y_{\mathrm{N}}} {F^k}^T$$

Cross (co)variance between kth and fth ifg:

$$Q_{\epsilon_{\mathrm{P}}^k \epsilon_{\mathrm{P}}^f} = Q_{e_{\mathrm{P}}^k e_{\mathrm{P}}^f} - F^k Q_{y_{\mathrm{N}} e_{\mathrm{P}}^f} - Q_{e_{\mathrm{P}}^k y_{\mathrm{N}}} {F^f}^T + F^k Q_{y_{\mathrm{N}}} {F^f}^T$$

1. With the first equation we can compute the variance of any target in any interferogram
2. With the second equation, we can compute the covariance between any two phase values in the time/space domains
3. F matrices captures all the filtering settings (e.g., type of the kernels, location of points, functional models, etc)

InSAR Generic Noise Model: precision of filtered phases

(Co)variance matrix of phase values of points P in kth ifg:

$$Q_{\epsilon_{\mathrm{P}}^k} = Q_{e_{\mathrm{P}}^k} - F^k Q_{y_{\mathrm{N}} e_{\mathrm{P}}^k} - Q_{e_{\mathrm{P}}^k y_{\mathrm{N}}} {F^k}^T + F^k Q_{y_{\mathrm{N}}} {F^k}^T$$

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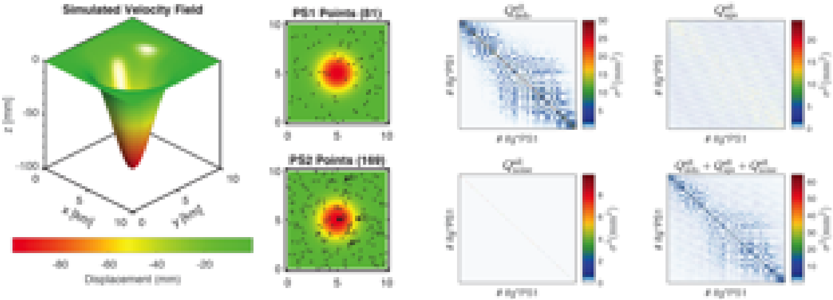
2. With the second equation, we can compute the covariance between any two phase values in the time/space domains
3. F matrices captures all the filtering settings (e.g., type of the kernels, location of points, functional models, etc)
4. Q matrices captures all the initial noise structure and effects of pre-processing (e.g., phase linking, multilooking, etc)

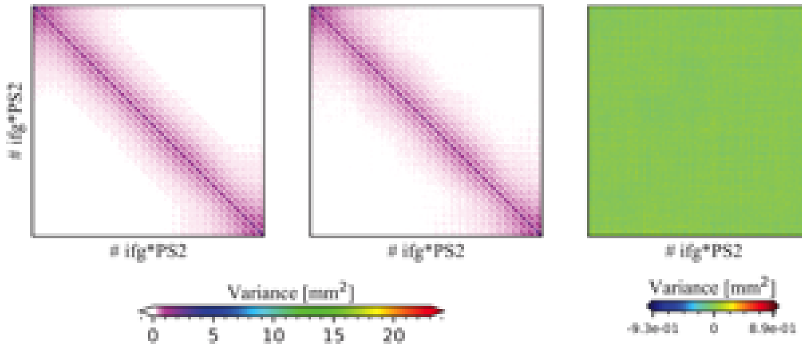
Synthetic data validation

To check the validity of the deriviations



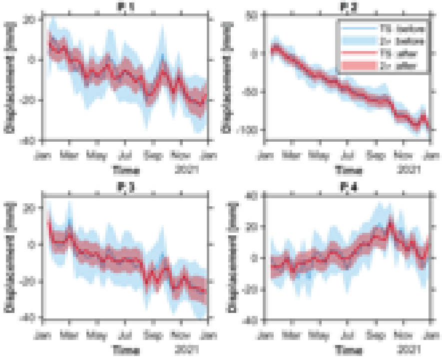
To check the validity of the deriviations

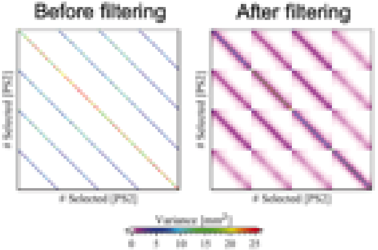
A: Proposed model B: Empirical Cov Matrix Difference $A - B$



InSAR Generic Noise Model: Synthetic Data Example

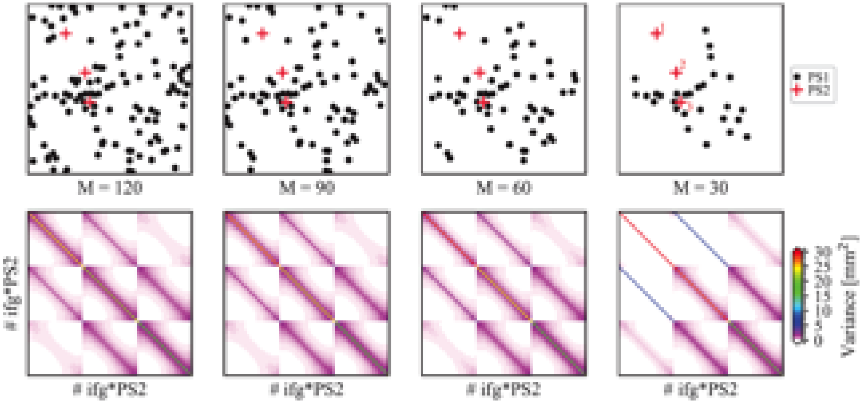
Effect of filtering





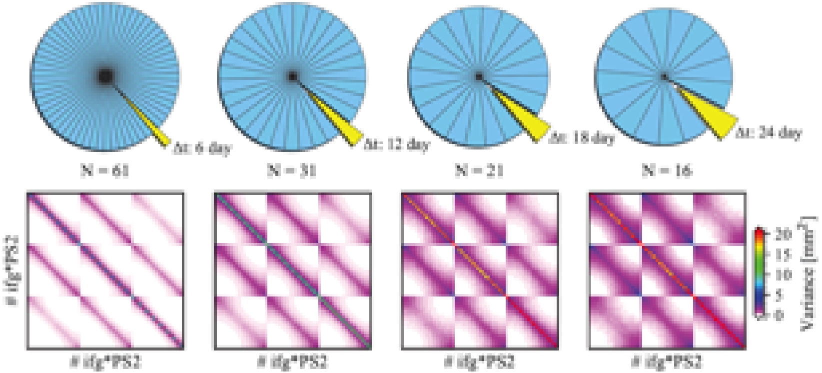
InSAR Generic Noise Model: Synthetic Data Example

Effect of point density of the initial network N



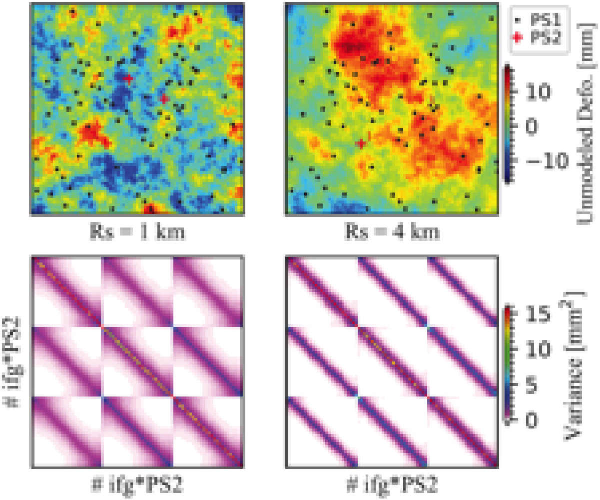
InSAR Generic Noise Model: Synthetic Data Example

Effect of revisit time



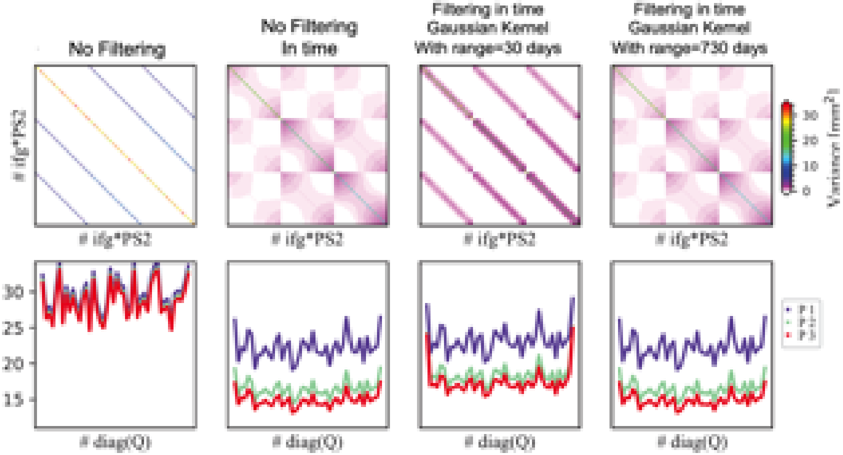
InSAR Generic Noise Model: Synthetic Data Example

Effect of unmodeled deformation



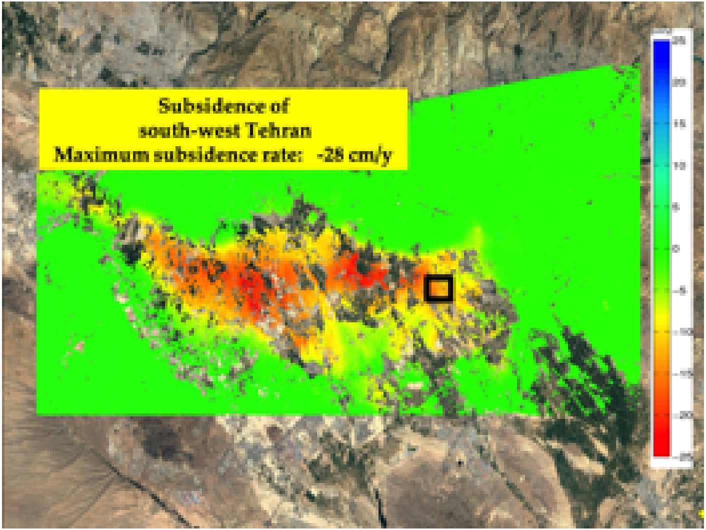
InSAR Generic Noise Model: Synthetic Data Example

Effect of Kernel type used for filtering in the time domain



InSAR Generic Noise Model: Real Data

30 SLC images, Sentinel-1A, one year of data 2021



InSAR Generic Noise Model: Real Data

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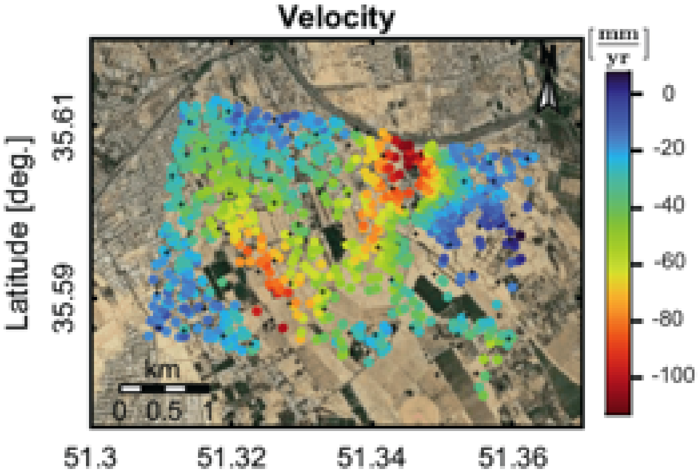

InSAR Generic Noise Model: Real Data

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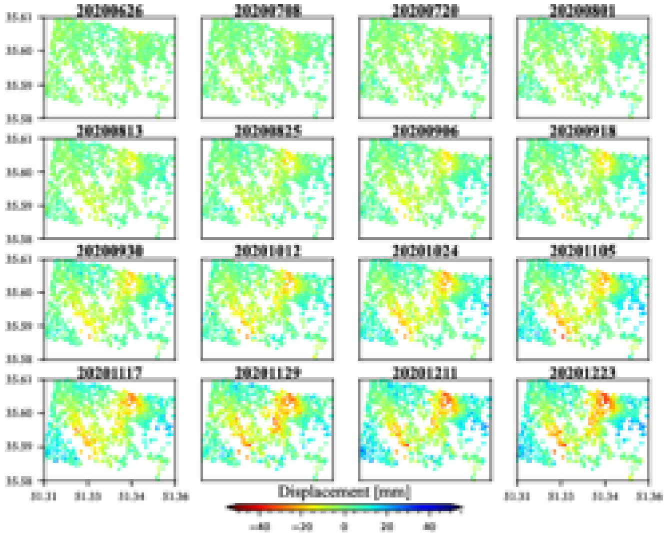

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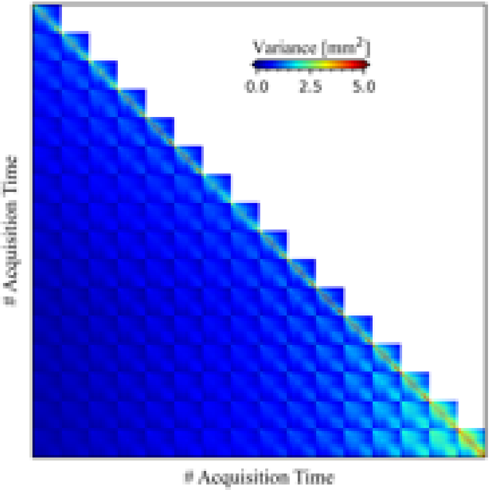

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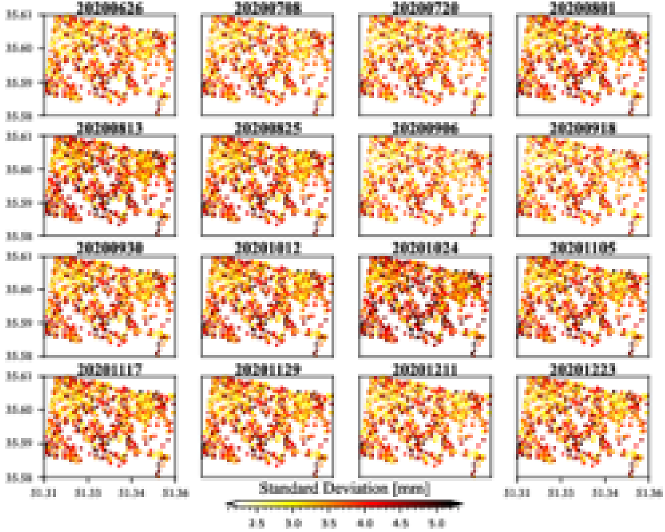

InSAR Generic Noise Model: Real Data

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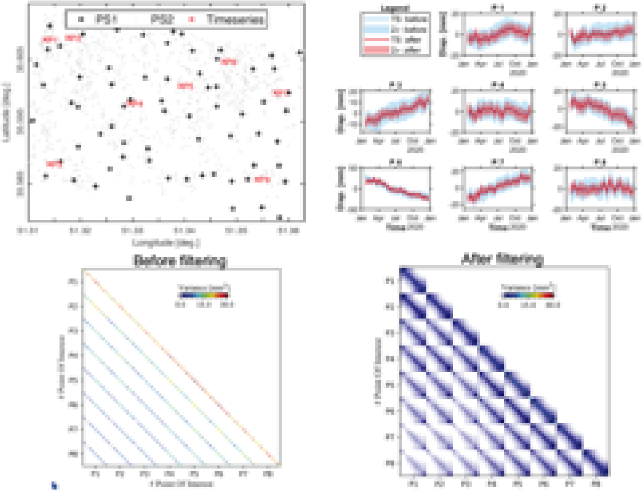

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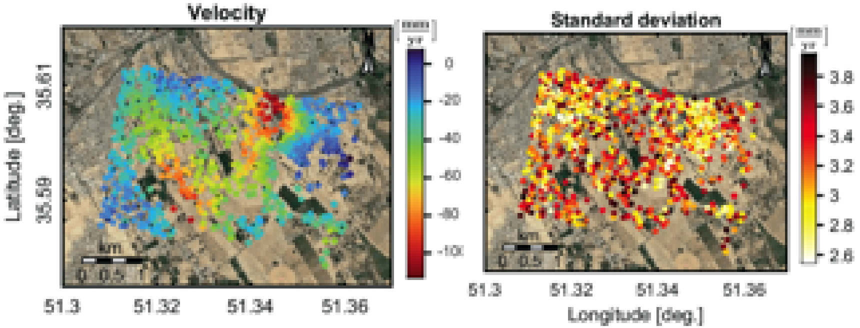
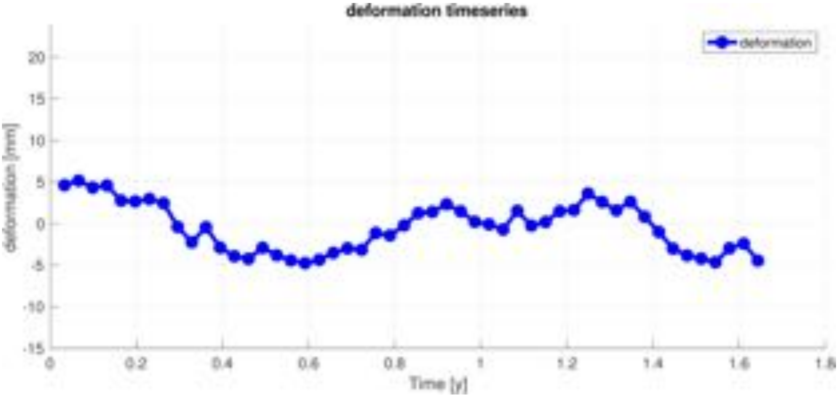


Importance of Noise Models for InSAR

Interpretation: Deformation or Noise?
Example 1:



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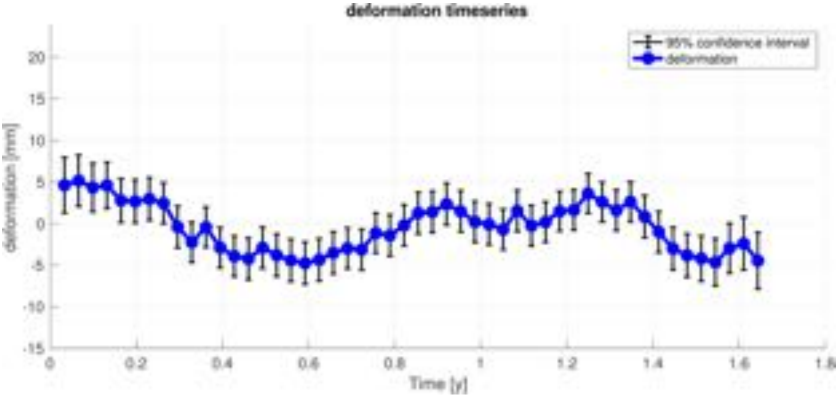

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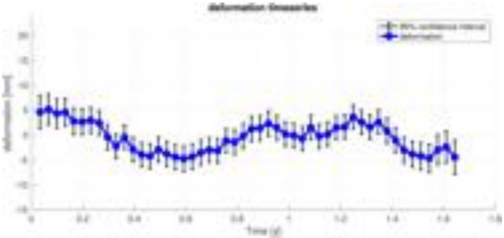
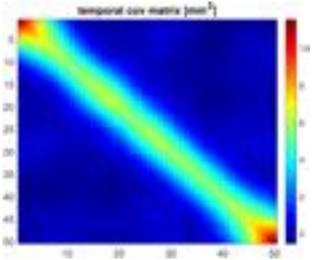




If we have the full Q, We can test whether this is a signal or noise:

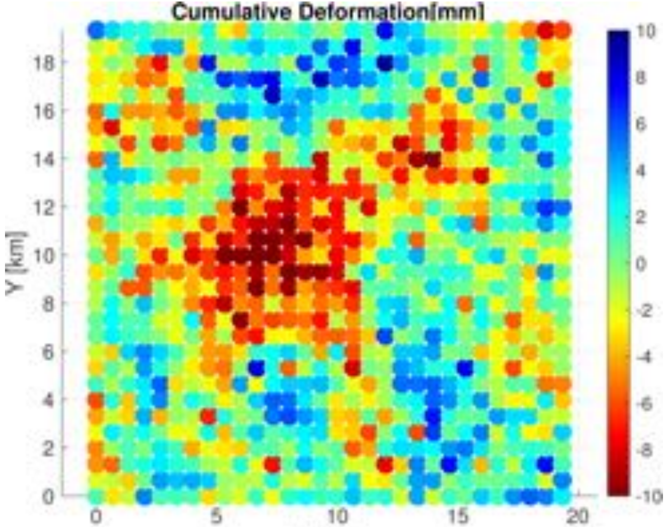
$$y^T Q^{-1} y \sim \chi^2(N) \qquad N: \text{ number of images}$$

$$y^T Q^{-1} y = 54 \quad \leq \quad \chi^2_{0.05}(50) = 67$$

Importance of Noise Models for InSAR

Interpretation: Deformation or Noise?

Example 2:



Importance of Noise Models for InSAR

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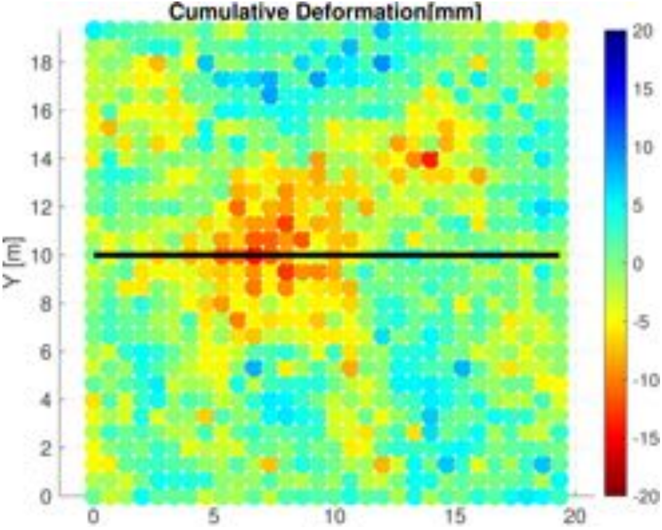

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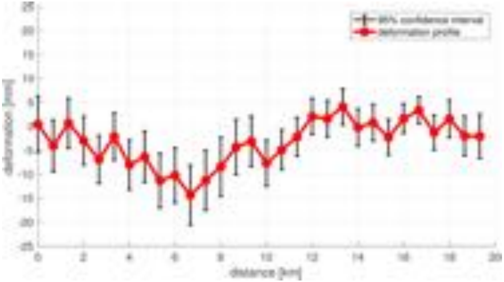

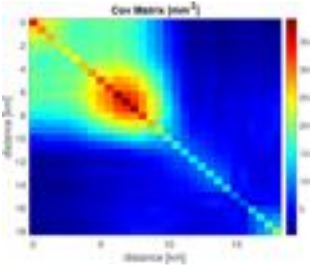

If we have the full Q, We can test whether this is a signal or noise:

$$y^T Q^{-1} y \sim \chi^2(N) \qquad N: \text{ number of points on the profile}$$

$$y^T Q^{-1} y = 34 \quad \leq \quad \chi^2_{0.05}(30) = 44$$

Conclusions

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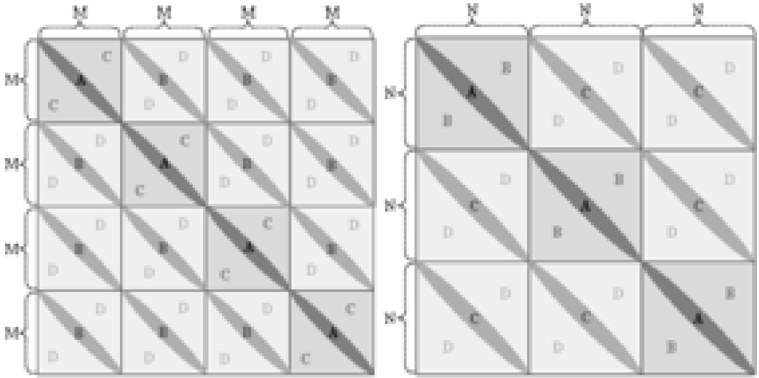
- The proposed formulation can be easily extended to cover other processing steps
- In TInSAR results we usually have spatio-temporally correlated/smooth noise
- Error-bars are not enough, we need to address the covariances!

Temporal vs Spatial Structure



(a) Temporal Structure (b) Spatial Structure

Fig. 4: The schematic presentation of the spatial-temporal structure of the covariance matrix for 4 points and 3 interferograms. The variance and covariance values can be classified into four different groups A: variances, B: spatial covariances, C: temporal covariances, and D: spatio-temporal covariances. a) Temporal structure b) Spatial structure.